Automatic Enumeration of Grid Classes

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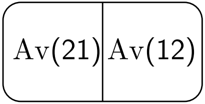
Joint work with Christian Bean, Jay Pantone and Henning Ulfarsson

Permutation Patterns 2018

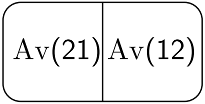
Grid class

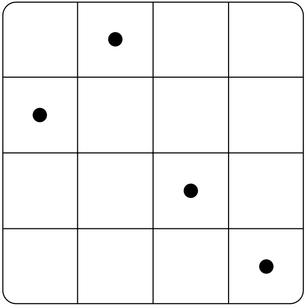
Given a matrix $\mathcal{M}$ whose entries are permutation classes, the permutations in the *grid class* defined by $\mathcal{M}$, Grid($\mathcal{M}$), are those which can have a grid drawn on it such that the subpermutation in each box is in the corresponding permutation class in $\mathcal{M}$.

Grid class



Grid class





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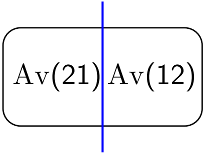

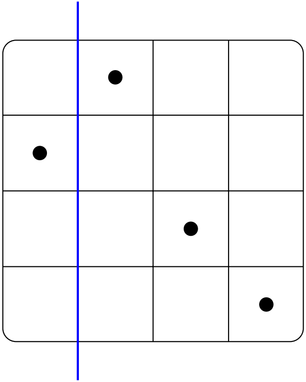

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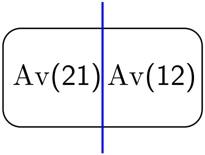

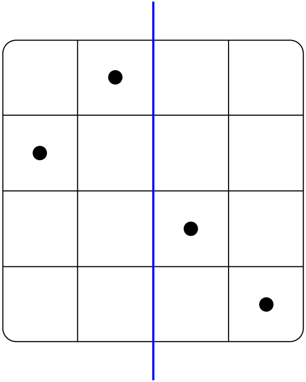

Grid class to Tilings

Theorem

Let $\mathcal{G}$ be a $1 \times N$ grid class. There exists a disjoint union of tilings $\mathcal{T}$ such that the gridded permutations in $\mathcal{T}$ are in bijection with the griddable permutations in $\mathcal{G}$.

Disambiguation of a grid class

$\mathrm{Av}(12|12|12)$

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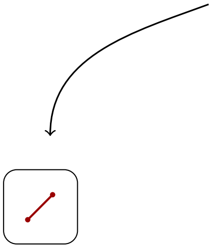

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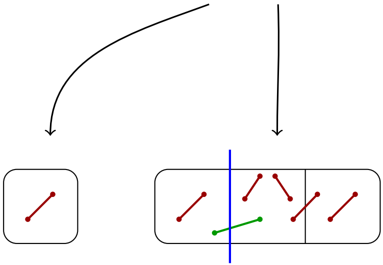

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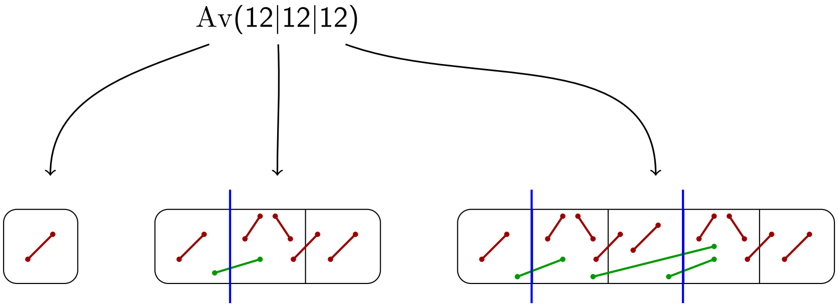

Disambiguation to combinatorial specification

$\mathrm{Av}(123, 132|12)$

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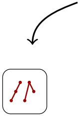

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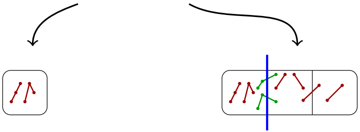

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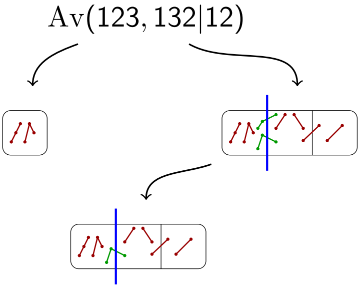

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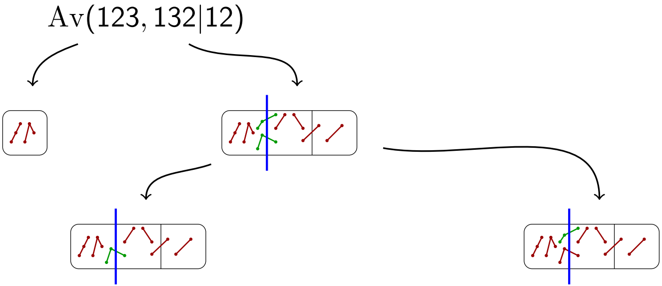

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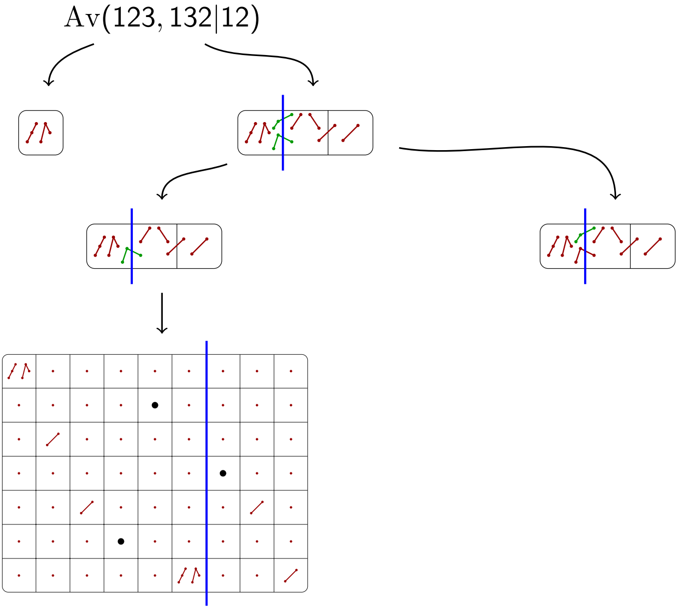

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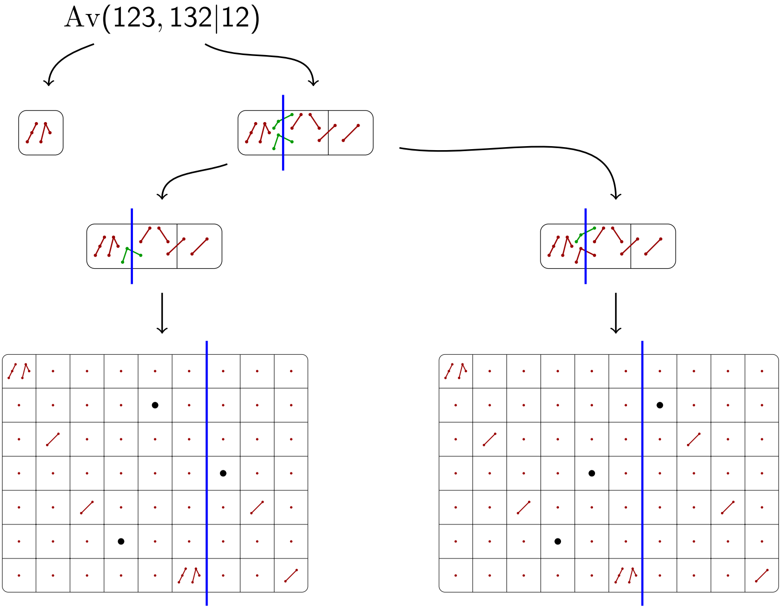

Av(123, 132|12)

$$F_0(x) = F_1(x) + F_2(x)$$
$$F_1(x) = \frac{1-x}{1-2x}$$
$$F_2(x) = F_3(x) + F_4(x)$$
$$F_3(x) = \frac{x^3}{(1-x)^2(1-2x)} \cdot F_1(x) \cdot F_1(\frac{x}{1-x})$$
$$F_4(x) = \frac{x^3}{(1-x)(1-2x)^2} \cdot F_1(x) \cdot F_1(\frac{x}{1-x})$$

Solving this system of equations gives us the generating function for the class is

$$F_0(x) = \frac{3x^4 - 15x^3 + 17x^2 - 7x + 1}{12x^4 - 28x^3 + 23x^2 - 8x + 1}$$

Results

We have ran the TileScope algorithm on all grid classes of the form $\mathrm{Av}(A|B)$ where A and B are subsets of $\mathcal{S}_2 \cup \mathcal{S}_3$. We only considered grid classes which are lexicographically minimum down to symmetry and where at least one side is not a finite class, in total there are 1100 grid classes we considered.

$n|m$	All	Success
1|1	20	7
2|1	72	27
2|2	87	74
3|1	80	36
3|2	172	171
4|1	60	27
5|1	24	15
6|1	4	3
other	581	581
Total	1100	941

Notable successes

We have managed to automatically enumerate all* of the juxtapositions from Brignall and Sliacan [1].

Juxtaposition	Minimal polynomial
Av(12|312) Av(12|321)	$-1 + x \cdot (x-2) \cdot F(x)^2 + (x+1) \cdot F(x)$
Av(12|213) Av(12|231)	$x \cdot (x-1)^2 \cdot F(x)^4 - (x-1)^2 \cdot F(x)^3 + (3 \cdot x - 2) \cdot (x-1) \cdot F(x)^2 + F(x) \cdot (x-1) + x$
Av(12|132) Av(12|123)*	$(x-1)^2 \cdot x^5 \cdot F(x)^4 - 2 \cdot x^3 \cdot (4 \cdot x - 1) \cdot (x-1)^2 \cdot F(x)^3 + x \cdot (x-1) \cdot (2 \cdot x^4 + 15 \cdot x^3 - 28 \cdot x^2 + 10 \cdot x - 1) \cdot F(x)^2 - (4 \cdot x - 1) \cdot (x-1) \cdot (2 \cdot x^3 - x^2 - 4 \cdot x + 1) \cdot F(x) + x^5 + 4 \cdot x^4 - 21 \cdot x^3 + 25 \cdot x^2 - 9 \cdot x + 1$

Av(12|213)

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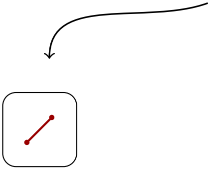

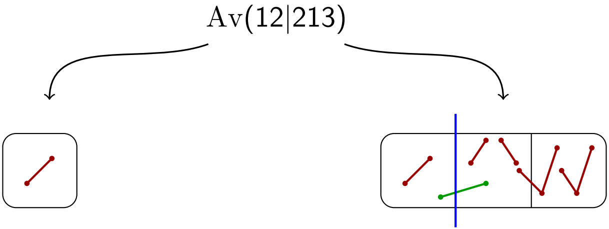

Av(12|213)

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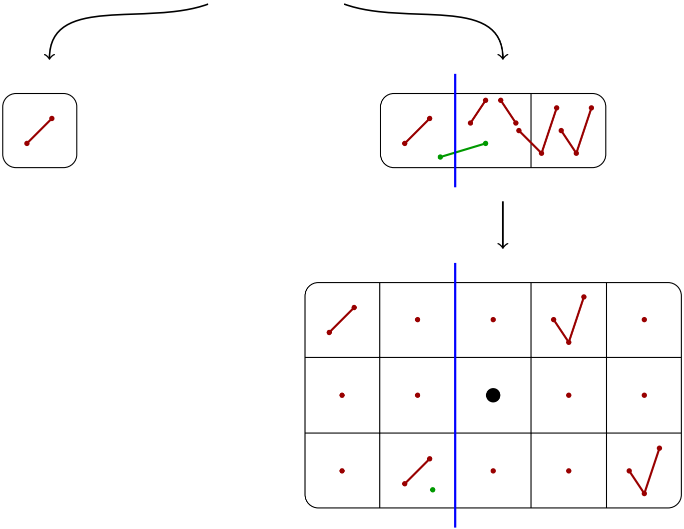

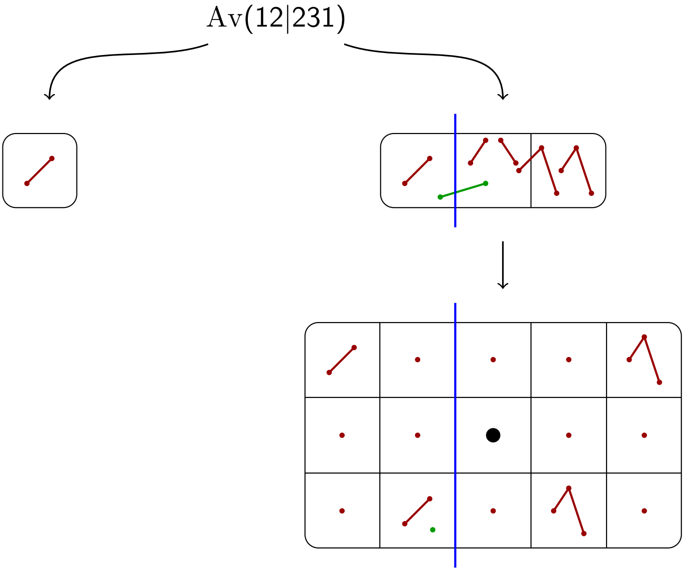
Av(12|231)

Non regular insertion encoding

$n \| m$	All	Success
1|1	18	5
2|1	56	11
2|2	15	5
3|1	60	16
3|2	16	15
4|1	45	12
4|2	10	10
5|1	18	9
5|2	2	2
6|1	3	2
Total	243	87

Fusion

$n \mid m$	All	Success
$1 \mid 1$	18	7
$2 \mid 1$	56	41
$2 \mid 2$	15	5
$3 \mid 1$	60	42
$3 \mid 2$	16	15
$4 \mid 1$	45	32
$4 \mid 2$	10	10
$5 \mid 1$	18	16
$5 \mid 2$	2	2
$6 \mid 1$	3	3
Total	243	173

Open questions

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If you have any interesting juxtapositions or $1 \times N$ grid classes you can fill out the form at `http://bit.ly/basisrequests`